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MODIFIED GRAVITY AS A COMMON CAUSE FOR COSMIC ACCELERATION AND FLAT GALAXY ROTATION CURVES

Abstract. Nowadays, relativistic theory is very popular among natural scientists.This is evidenced by the large number of articles published in recent years. Based on the results of these works, one can be convinced that a huge work is being done in the study of the universe from a theoretical point of view.In this work approximate simulations of the redistribution of dark matter, in different proportions to elliptical orbits with different radii was studied. It was shown that the rotation velocity of objects in a spiral galaxy is dependent on the enclosed mass at any given radius. The enclosed mass is comprised of the mass of the stars, the mass of the gas, and the mass of the dark matter. The theory provided in this work assumes that the law of gravity is different from the usual Newtonian form.

Keywords: dark matter, the law of gravity, the enclosed mass, the speed of rotation of objects, the Galaxy.

Introduction. Natural science is now in the beginning of a new, extremely interesting stage in his development. It is not able primarily to the fact that the science of the microcosm – the physics of elementary particles and the science of the universe –cosmology – the science become one of the fundamental properties of the surrounding world. Different methods they respond to the samequestions: what the Universeis filled with matter today? What wasevolutionin the past? What are the processesthat took placebetween elementary particlesin the early universe, ledeventuallyto itscurrent state? If arelatively recentdiscussion of such matters stopped at the level of hypotheses, but today there are numerous experimental and observational data toobtain quantitative. This isanother feature ofthe current stage: cosmology in the last 10 – 15yearshas becomean exact science. Already, the dataof observational cosmologyare of high accuracy, even more information about thedate andthe early universewill be receivedin the coming years [1].

There are strong arguments infavor of the factthat much of thematter in the universedoes notradiateand thereforeinvisible. The presence ofthisinvisible mattercan be recognized by its gravitational interaction with the radiating matter. The study of clusters of galaxies and galactic rotation curves indicates the existence of this so-called dark matter. So, by definition, dark matter – a matter which does not interact with electromagnetic radiation, that is, it does not emit or absorb. Dark matter has been introduced to explain many independent gravitational effects at different astronomical scales, in galaxies, groups of galaxies, clusters, super clusters and even across the full horizon [2].

The first detection of invisible matter dates back to last century. In 1844, Friedrich Bessel, in a letter to Karl Gauss wrote that unexplained irregularity in the motion of Sirius may be the result of its gravitational interaction with some neighboring body, the latter in this case must have a sufficiently large mass. At the time of the Bessel this dark companion of Sirius was invisible, itsoptically detected only in 1862. He was a white dwarf, called Sirius – B, while Sirius was named Sirius – A [3].

Method and theory. Flat galaxy rotation curves and the accelerating Universe both imply the existence of a critical acceleration, which is of the same order of magnitude in both the cases, in spite of the galactic and cosmic length scales being vastly different. Yet, it is customary to explain galactic acceleration by invoking gravitation-ally bound dark matter, and cosmic acceleration by invoking a `repulsive` dark energy.

Dark matter has been postulated to explain the at non – Keplerian behavior of rotation curves of galaxies, and the high dispersion velocities of galaxy clusters. Dark energy/cosmological constant have been postulated as an explanation for the observed late-time cosmic acceleration. Together, cold dark matter and a cosmological constant provide the standard CDM model of structure formation and large-scale-structure which best fits the current cosmological data. Nonetheless, there are chinks in the amour! Apart from the well-known facts that dark matter has not yet been detected in the laboratory, and that the inferred value of the cosmological constant is too low compared to the theoretically preferred value by some 10^{120} orders of magnitude, there is another remarkable observation which the CDM model does not seem to account for. And that is the existence of a universal acceleration $cH_0 \sim 10^{-10}$ cm sec^{-2}, where H_0 is the present value of the Hubble parameter, which prevails in the outer regions of galaxies, in galaxy clusters, and at the edge of the observed Universe:

$$\frac{GM_{galaxy}}{R^2_{galaxy}} \sim \frac{GM_{cluster}}{R^2_{cluster}} \sim \frac{GM_{Universe}}{R^2_{Universe}} \sim cH_0 \tag{1}$$

We seek an understanding of the cause for this universal acceleration, and for at galaxy rotation curves and cosmic acceleration, by postulating a fourth order modified gravity as a common explanation, as an alternative to dark matter and dark energy. We propose, for reasons to be discussed below, that the current structure and evolution of the Universe is described by the following e effective field equations:

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{8\pi G}{c^4} T^{\mu\nu} + L^2 R^{\mu\nu\alpha\beta}_{;\alpha\beta} , \tag{2}$$

The length parameter L is scale-dependent and defined as being proportional to the square-root of the mass M of the system under study, in such a way that $GM=L^2$ is a universal constant equal to cH_0. This holographic assumption [mass proportional to area rather than volume] seems to be universally valid for large astrophysical systems and entirely consistent with the observation.

We will solve these modified gravity equations for two different space- time metrics:

(i) The Newtonian weak- field non-relativistic approximation for the scalar potential ϕ:

$$ds^2 = (1+\frac{2\phi}{c^2})c^2dt^2 - dx^2 - dy^2 - dz^2; \tag{3}$$

(ii) The spatially flat Robertson – Walker cosmological metric:

$$ds^2 = c^2dt^2 - a^2(t)\left[dx^2 + dy^2 + dz^2\right]. \tag{4}$$

In the first case we will show how the resulting fourth order biharmonic modification of the Poisson equation explains at galaxy rotation curves without dark matter, and in the second case we will show how a fourth order modification of the Friedmann equations explains late-time cosmic acceleration without dark energy.

It can be shown that for the weak- field metric (3) the modified equations (2) reduce to the following fourth order biharmonic correction to the Poisson equation (2; 3)

$$\nabla^2\phi - L^2\nabla^4\phi = 4\pi G\mu(r) \tag{5}$$

where $\mu(r)$ is the matter density distribution of interest. From observations of galaxies this is known to be

$$\mu(r) = \frac{3}{4\pi r^3}\beta M(r)\left[\frac{r_c}{1+r_c}\right] \tag{6}$$

where

$$M(r) = 4\pi\int_0^r dr' r'^2 \mu(r') = M\left(\frac{r_c}{1+r_c}\right)^{3\beta} \tag{7}$$

and

$$\beta = \begin{cases} 1 \text{ for HSB galaxies,} \\ 2 \text{ for LSB \& Dwarf galaxies.} \end{cases} \tag{8}$$

For this mass distribution, the biharmonic equation (5) can be solved to obtain the following remarkable Yukawa form for the radial component of the gravitational acceleration a = - $\nabla\phi$ inside the galaxy

$$\alpha(r) = -\frac{GM(r)}{r^2}\left\{1+\sqrt{\frac{M_0}{M}}\left[1-\exp\left(-\frac{r}{L}\right)\left(1+\frac{r}{L}\right)\right]\right\} \tag{9}$$

where M_0 is a constant of integration and M is the total mass of the galaxy [4]. The rotation curve $v(r)$ of the galaxy is obtained by setting $a(r) = v^2(r)/r$. If we choose L according to the prescribed relation $GM/L^2 = cH_0$ [this gives L∼10 kpc] and if we choose $M_0 = 10^{12}\, M_\odot$ it is possible to t the at rotation curves of a large class of galaxies, without invoking dark matter [4, 5]. It is to be noted that the values of L and M_0 chosen here are consistent with all known laboratory and astronomical tests of departures from the inverse square law [6]. For r≪ L, the inverse square law holds. Also, for r ≫ L the inverse square law holds but with an e effectively larger value of the gravitational constant. Interesting new physics arises around r≳ L.

Greater insight into the modified acceleration obtained by us can be had by expanding the acceleration (9) around r = L and writing it as a sum of two terms: a part that falls as $1/r^2$ and is independent of L, and a part that falls as 1/r and depends on L. This gives

$$\alpha(r) \approx -\frac{GM(r)}{r^2}\left[1+\sqrt{\frac{M_0}{M}}\left\{1-\frac{3}{e}\right\}\right]-\frac{GM(r)}{\kappa}\left[\sqrt{\frac{M_0}{M}}\frac{1}{L_e}\right]. \tag{10}$$

The second term dominates for r ≳ L. Fig. 1 above plots the rotation velocity curve, and it is clear that the Keplerian fall – off due to conventional Newtonian acceleration [first term] is modified to a at rotation curve [second term] for r ≳ L, because of the proposed generalization of Einstein gravity.

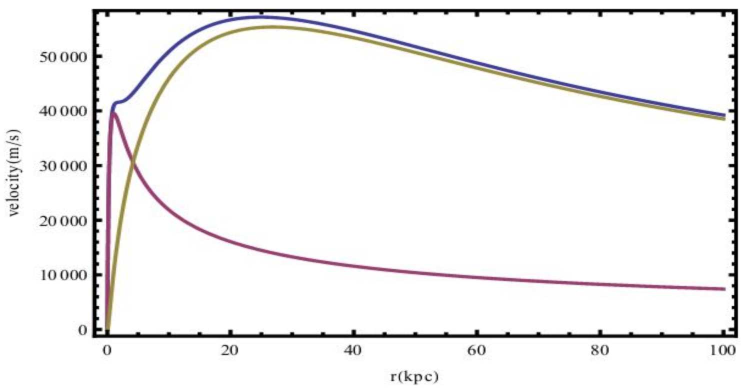

Figure - 1. Red curve: velocity due to first term, Green curve: velocity due to second term, Blue curve: total velocity

Next, we turn to cosmology and write the modified Friedmann equations resulting from Eqn. (2), for the Robertson-Walker metric. Since one is no longer in the weak- field limit, the equations are complicated, but it is adequate for our purpose to write them in the form

$$\frac{\dot{a}^2}{a^2}-L_U^2F_1\left(a,\dot{a},\ddot{a},\dddot{a}\right)=\frac{8\pi G}{3}\rho \tag{11}$$

$$\frac{2\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}-L_U^2F_1\left(a,\dot{a},\ddot{a},\dddot{a},\dot{a}\right)=8\pi G\rho \tag{12}$$

where F_1 and F_2 are known functions of their arguments which we do not write explicitly. In accordance with the scaling principle, since we are now considering the entire observable Universe, we have $L_U = cH_0^{-1}$. It turns out that these equations possess a very interesting analogy with the weak- field case considered above. There is a space ↔time symmetry: the behaviour seen in the previous case around r ≳ L is repeated here for $t \geq H_0^{-1}$.

For times t ≪ L_U the modifying gravity terms can be neglected and these equations reduce to the standard Friedmann equations which give a decelerating solution. This is analogous to the behaviour seen in the solution (9) for the gravitational acceleration: just as the inverse-square law holds for r L in Eqn. (9), for $t \langle\langle H_0^{-1}$ one recovers the standard Friedmann evolution. Things start to get interesting for t ≳ $L_U = H_0^{-1}$. For the sake of simplification one can divide the evolution into two phases: (i) evolution for $t \langle\langle H_0^{-1}$, and (ii) evolution for $t \geq H_0^{-1}$. In the second case, one can show from Eqn. (12) for the dust case p = 0, assuming a power-law solution, that the evolution is dominated by the modifying terms, and the scale-factor is given by

$$a(t) = \left(\frac{t - L_U}{T - L_U} \right)^{3/2} \tag{13}$$

where T is the current age of the universe. One has an accelerating solution with $\ddot{a} \rangle 0$. This is in complete analogy with the galactic case: the modifying term increases the acceleration, and in the present cosmological case the rise is enough to change the sign from deceleration to positive acceleration. Furthermore, the acceleration decreases with increasing time, and it can be shown that for t≫H_0^{-1} the decelerating phase reappears. It is easily shown that T = $5H_0^{-1}$=2, and as can be expected the acceleration is of the order of cH_0. [The vanishing of the scale factor at t = L_U is only an approximation, and simply reflects the smallness of its value at t = L_U compared to the present value a = 1; the true singularity of course occurs at the beginning of the previous decelerating phase].

Using the solution (13) for the scale factor in the first Friedmann equation (11) it can be shown that the dust density evolves as

$$\rho(t) \infty \frac{1}{t^2} \infty \frac{1}{a^{4/3}} \tag{14}$$

The density falls with the scale factor slowly compared to the standard case [where $\rho \infty 1/a^3$], because the modifying term in the Einstein equation implies that

$$\left(\frac{8\pi G}{c^4} T^{\mu\nu} + L^2 R^{\mu\nu\alpha\beta}_{;\alpha\beta} \right)_{;\nu} = 0 \tag{15}$$

The onset of fourth order gravity at the present cosmological epoch, which we have proposed as a phenomenological representation of an underlying physics, implies an exchange of energy-momentum between gravity and matter fields at galactic and cosmological scales. This is one possible unified way of understanding the otherwise unusual features of at rotation curves and cosmic acceleration.

The underlying physics may possibly have to do with the effect that averaging over small-scale in homogeneities has on dynamics on larger scales, a phenomenon which is perhaps not fully understood at present. In fact, such considerations led to the suggestion from Szekeres [7] of essentially the same equation as Eqn. (2) proposed here.

Understanding structure formation in this picture is a challenge. But we have a unified description of critical acceleration on different scales, which comes, not from invoking different matter/energy components, but a phenomenological modification of gravity whose root cause may be the same at different scales, and which calls for further study.

Results. The results presented above lead to the following description of galaxy evolution. The galaxies form to a large mass early on. While the majority of galactic nuclei are active (2 to 5 BY after the Big Bang), galaxies redistribute their dark matter. During the same period, the visible matter unfolds from compact orbits to larger orbits (a few times larger, depending on M/L ratios). When the redistribution of matter is complete (5 to 7 BY after the Big Bang, and in a timeframe of ~1 BY for any particular galaxy), the galaxies have transformed from compact structures into highly organized spiral structures. These galaxies have acquired correlated dark matter and visible matter components; have grown in size but not mass (although the M/L ratio in the visible part of the galaxy has decreased significantly); and, have acquired 'flat' rotation curves. They appear similar to those observed in the work [8]. Thereafter, for the next ~7 to 9 BY, if they are in a crowded space, then they undergo further development by collisions and mergers (including further growth in size and mass, and the formation of elliptical galaxies from equal mergers); or, if they are isolated, then they evolve on their own.

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ҒАРЫШТЫҚ ҮДЕУ МЕН ГАЛАКТИКАЛАРДЫҢ ЖАЗЫҚ АЙНАЛУ ҚИСЫҚТАРЫНЫҢ ЖАЛПЫ СЕБЕБІ РЕТІНДЕ ТҮРЛЕНДІРІЛГЕН ГРАВИТАЦИЯ

Аннотация. Релятивистік теория қазіргі кезде жаратылыстану зерттеуші ғалымдары арасында өте танымал. Оған соңғы жылдары жарық көрген мақалалардың көптігі дәлел. Осы жұмыстардың нәтижелеріне сүйене отырып, Әлемді теориялық тұрғыдан зерттеу бойынша орасан зор жұмыс жүргізіліп жатқандығына сенімді болуға болады. Бұл жұмыста қараңғы материяны әртүрлі пропорцияларда әр түрлі радиусы бар эллиптикалық орбиталарға қайта бөлудің шамамен модельдеуі зерттелді. Спиральды галактикадағы заттардың айналу жылдамдығы кез-келген радиуста ондағы массаға байланысты екендігі көрсетілген. Қорытынды масса жұлдыздардың массасынан, газдың массасынан және қара материяның массасынан тұрады. Бұл жұмыста ұсынылған Теория гравитация заңы кәдімгі Ньютон формасынан өзгеше деп болжайды.

Тірек сөздер: қараңғы зат, тартылыс заңы, қорытынды масса, заттардың айналу жылдамдығы, Галактика.

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МОДИФИЦИРОВАННАЯ ГРАВИТАЦИЯ КАК ОБЩАЯ ПРИЧИНА КОСМИЧЕСКОГО УСКОРЕНИЯ И ПЛОСКИХ КРИВЫХ ВРАЩЕНИЯ ГАЛАКТИК

Аннотация. В наши дни релятивистская теория очень популярна среди ученых в области естественных наук. Об этом свидетельствует большое количество статей, опубликованных за последние годы. По результатам этих работ можно убедиться, что проводится огромная работа по изучению Вселенной с теоретической точки зрения. В этой работе было изучено приближенное моделирование перераспределения темной материи в разных пропорциях на эллиптические орбиты с разными радиусами. Было показано, что скорость вращения объектов в спиральной галактике зависит от массы тела на любом заданном радиусе. Замкнутая масса состоит из массы звезд, массы газа и массы темной материи. Теория, представленная в этой работе, предполагает, что закон всемирного тяготения отличается от обычной ньютоновской формы.

Ключевые слова: темная материя, закон тяготения, заключенная масса, скорость вращения объектов, Галактика.